Solution by CHRISTIAN HORNUNG, A. M., Heidelberg University, Tiffin, Ohio.

Let a be the population at the beginning; then at the end of the first year the population will be $a+(\frac{1}{30}-\frac{1}{50})a=\frac{76}{75}a$; at the end of the second year $\frac{76}{75}$ of $\frac{76}{75}a=(\frac{76}{75})^2a$; and at the end of the nth year $(\frac{76}{75})^na$.

Hence $(\frac{76}{75})^na=3a$, whence $n=\dfrac{\log 3}{\log 76-\log 75}=82.94$, or nearly 83 years.

Also solved by J. Scheffer, M. E. Graber, F. R. Honey, R. D. Carmichael, A. H. Holmes, G. W. Greenwood, L. E. Newcomb, and the Proposer.

233. Proposed by J. T. KEYES, Fogg High School, Nashville, Tenn.

At what time between 10 and 11 o'clock is the second hand of a clock one minute space nearer to the hour hand than it is to the minute hand?

I. Solution by J. SCHEFFER, Hagerstown, Md.

Let the time be x minutes past 10 o'clock. We assume that at the beginning of every minute the second hand points at 12 on the dial. The distance of the second hand at the required time is $60x-x=59x$; and that of the second hand from the hour hand is $60-60x-(10-\frac{1}{12}x)=50-60x+\frac{1}{12}x$.

$\therefore 59x=51-60x+\frac{1}{12}x$, whence $x=\frac{612}{1427}$ minutes$=25\frac{1045}{1427}$ seconds.

II. Solution by G. W. GREENWOOD, M. A., Professor of Mathematics, McKendree College, Lebanon, Ill.

Let us measure the spaces clockwise from the minute hand to the second hand, and from the second hand to the hour hand. At n minutes and t seconds after ten, the number of minute spaces after the hour at which the hour hand, minute hand, and second hand stand are, respectively,

$$50+\frac{n}{12}+\frac{t}{720},\quad n+\frac{t}{60},\quad t.$$

$$\therefore 50+\frac{n}{12}+\frac{t}{720}-t+1=t-\left(n+\frac{t}{60}\right);\ i.\ e.,\ 36720+780n=1427t.$$

By putting $n=1, 2, \ldots\ldots, 59$, we get the corresponding values of t.

Also solved by Frederic R. Honey, A. H. Holmes, and L. E. Newcomb.

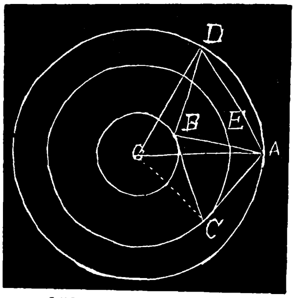
GEOMETRY.

257. Proposed by G. I. HOPKINS, A. M., Manchester, N. H.

Construire un triangle équilatéral sachant qu'el doit s'appuyer par ses trois sommets sur trois circonferences concentriques données. *Rouché et Comberousse.*

Solution by J. SCHEFFER, A. M., Hagerstown, Md.

Draw any radius OA of the largest circle, and on this construct the equilateral triangle OAD. From D as center with a radius OE of the middle circle draw an arc cutting the smallest circle at B, so that $DB=OE$, and draw BA, make $AC=AB$, and connect C with B, then ABC be the required triangle.



For, in the two triangles ABD and ACO, $AD{=}AO$, $BD{=}OC$, $AB{=}AC$.

$\therefore \triangle ABD{=}\triangle ACO$; $\therefore \angle DAB{=}\angle OAC$.

$\therefore \angle DAO{=}\angle BAC{=}60^\circ$.

$\therefore \triangle ABC$ is an equilateral triangle having its vertices in the three concentric circumferences.

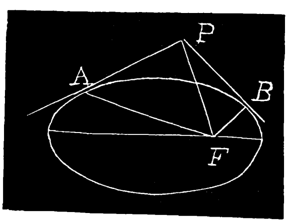
G. W. Greenwood called attention to the similarity of this problem to No. 250.

258. Proposed by B. F. FINKEL, A. M., Professor of Mathematics, Drury College, Springfield, Mo.

Prove that the tangents to an ellipse from any external point subtend equal angles at the focus, by means of the formula $\tan\phi{=}(m_1-m_2)/(1+m_1m_2)$, where ϕ is the angle between the focal radius of either of the points of tangency and the line joining the focus and the external point, and m_1 and m_2 are the slopes of these two lines.

Solution by J. SCHEFFER, A. M., Hagerstown, Md.

Designate the coördinates of the point of contact A by x_1, y_1, and those of the point of contact B, x_2, y_2, the equation of the ellipse being $a^2y^2+b^2x^2=a^2b^2$; then the equation of tangents PA and PB will be, respectively, $a^2yy_1+b^2xx_1=a^2b^2$, and $a^2yy_2+b^2xx_2=a^2b^2$. By solving these as two simultaneous equations we find



$$x=\frac{a^2(y_2-y_1)}{x_1y_2-x_2y_1},\quad y=-\frac{b^2(x_2-x_1)}{x_1y_2-x_2y_1},$$

which are the coördinates of the point P. Since the coördinates of focus F are $-ae$ and 0, we find the slope of PF

$$=-\frac{\dfrac{b^2(x_2-x_1)}{x_1y_2-x_2y_2}}{\dfrac{a^2(y_2-y_1)}{x_1y_2-x_2y_1}+ae}=-\frac{b^2(x_2-x_1)}{a[ay_2-ay_1+e(x_1y_2-x_2y_1)]},$$

and the slope of $AF=\dfrac{y_1}{x_1+ae}$.

$$\therefore \tan PFA=\left[-\frac{b^2(x_2-x_1)}{a[ay_2-ay_1+e(x_1y_2-x_2y_1)]}+\frac{y_1}{x_1+ae}\right]$$

$$\div\left[1-\frac{b^2y_1(x_2-x_1)}{a(x_1+ae)[ay_2-ay_1+e(x_1y_2-x_2y_1)]}\right]$$